

Supplement to Chapter 10

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Data Files

The data set available through the book's website contains randomly simulated dichotomous item responses of 3,750 examinees on 21 items. It was assumed that each of the 21 items is associated with one of 3 sub-areas (items 1 - 7 for area 1, items 8 - 14 for area 2, and items 15 - 21 for area 3). Furthermore, it was assumed that 3,750 examinees were from 30 schools. Schools sizes were varied as follows; 5 schools with 50 students, 10 schools with 100 students, 10 schools with 150 students, and 5 schools with 200 students. One dichotomous student characteristic variable was generated, such that half of students in each school display the characteristic. Item response data was generated in the way that the variable affected the probability of correct answer (DIF) for one of the 7 items in each sub-area (items 4, 11, and 18). Also, it was assumed the magnitudes of DIF were different between schools (small variation for item 4, medium variation for item 11, and large variation for item 18). For analyses with continuous measurement indicators (HLM and 2-level CFA approaches), three sub-area scores are used. Sub-area scores were derived by simply summing 7 items in each sub-area. Since the numbers of items are the same for the 3 sub-areas in this generated data, we retained the summed score, rather than the transformation described in the chapter (proportion of correct response \times 10). These item response data are in SPSS data file `Ch10_data.sav`. Based on this data file, several different data files have been prepared for different types of analysis. They are summarized in a table below.

Data file	Descriptions
HLM_MMM_L1.sav	Level-1 data file for HLM analysis in SPSS format
HLM_MMM_L2.sav	Level-2 data file for HLM analysis in SPSS format
HLM_MMM_L3.sav	Level-3 data file for HLM analysis in SPSS format
HGLM_L1.sav	Level-1 data file for HGLM analysis in SPSS format
HGLM_L2.sav	Level-2 data file for HGLM analysis in SPSS format
HGLM_L3.sav	Level-3 data file for HGLM analysis in SPSS format
Ch10Mplus.dat	Data file for 2-level CFA analysis for Mplus in tab-delimited text format

Three MDM template files for HLM software are provided:

MDM template file	Descriptions
HLM_MMM.mdmt	MDM template file for 3-level HLM analysis
MHLM_MMM.mdmt	MDM template file for multivariate HLM analyses
Ch10HGLM.mdmt	MDM template file for HGLM analyses

HLM software requires users to specify appropriate path in the MDM template files. They are prepared assuming the original folder structure is kept as C:\Ch10\ . . . Some ready-to-use syntax files are also provided.

Syntax file	Descriptions
TLPM.hlm	HLM syntax file for 3-level univariate HLM psychometric model
MVPM.hlm	HLM syntax file for multivariate HLM psychometric model
1_Uncond.hlm	HLM syntax file for unconditional HGLM analysis
2_Cond.hlm	HLM syntax file for conditional HGLM analysis
Ch10Mplus1.inp	Mplus syntax file for 2-level CFA with unit factor loadings
Ch10Mplus2.inp	Mplus syntax file for 2-level CFA with heterogeneous factor loadings 1
Ch10Mplus3.inp	Mplus syntax file for 2-level CFA with heterogeneous factor loadings 2

Note that this generated data set is different from the one used in the book. Therefore, results with this generated data set will be different from ones presented in the book. Some results from the generated data set are presented below.

HLM Analyses

1. Some Descriptive Statistics

21 items and three subscales ($n = 3$).

Subscale 1 --- 7 items

Subscale 2 --- 7 items

Subscale 3 --- 7 items

$N = 3750$ students, $K = 30$ schools. Thus, $J = 3750/30 = 125$ students/school.

Subtest level (Level 1) descriptive statistics (file: HLM_MMM_L1.sav):

Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation
subs_score	11250	0	7	3.69	1.781
dsub1	11250	0	1	.33	.471
dsub2	11250	0	1	.33	.471
dsub3	11250	0	1	.33	.471
Valid N (listwise)	11250				

Student level (Level 2) descriptive statistics (file: HLM_MMM_L2.sav):

Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation
area1	3750	0	7	3.66	1.775
area2	3750	0	7	3.70	1.803
area3	3750	0	7	3.71	1.764
sub1	3750	.00	7.00	3.6621	1.77494
sub2	3750	.00	7.00	3.7016	1.80284
sub3	3750	.00	7.00	3.7064	1.76370
total	3750	.00	21.00	11.0701	4.24033
Valid N (listwise)	3750				

School level descriptive statistics (file: HLM_MMM_L3.sav):

Descriptive Statistics

	N	Minimum	Maximum	Mean	Std. Deviation
subs1_mean	30	2.75	4.48	3.6854	.40503
subs2_mean	30	2.49	4.61	3.7362	.44952
subs3_mean	30	2.59	4.50	3.6999	.43920
total_mean	30	7.91	13.59	11.1214	1.15107
N_student	30	50	200	125.00	48.690
Valid N (listwise)	30				

Reliability

Reliability Statistics

Cronbach's Alpha	N of Items
.765	21

Standard error of measurement:

$$4.24033 \times \sqrt{1 - .765} = 2.056.$$

2. Results from Three-Level psychometric model: Comparable to Table 10.3.

a. Fixed effects

	Estimate	Standard error	t-ratio	df	p-value
γ_{000}	3.677	.072	50.900	29	< .001
γ_{100}	.039	.031	1.289	11247	.198
γ_{200}	.044	.031	1.445	11247	.148

b. Variance components

	Estimate	Standard error	df	Chi-Square	p-value	Reliability
Level 1						
σ^2	1.758	.029				
Level 2						
τ_π	1.274	.044	3720	11808.648	< .001	.685
Level 3						
τ_β	.130	.038	29	305.914	< .001	.881

c. Model Summary

Deviance	# of parameter estimated
42673.165	6

Student level (Level 2) reliability (corresponds to equation (10.5):

$$\hat{\alpha}_\pi = \frac{\hat{\tau}_\pi}{\hat{\tau}_\pi + \hat{\sigma}^2 / n} = \frac{1.274}{1.274 + 1.758 / 3} \cong 0.684. \text{ (HLM output = .685)}$$

ICC:

$$corr(\hat{\pi}_{0,jk}, \hat{\pi}_{0,j'k}) = \frac{\hat{\tau}_\beta}{\hat{\tau}_\beta + \hat{\tau}_\pi} = \frac{.130}{.130 + 1.274} \approx 0.093.$$

School level (Level 3) reliability:

$$\alpha_\beta = \frac{1}{K} \sum \hat{\alpha}_{\beta k} \text{ where } \hat{\alpha}_{\beta k} = \frac{\hat{\tau}_\beta}{\hat{\tau}_\beta + \hat{\tau}_\pi / J_k + \hat{\sigma}^2 / J_k n}.$$

In the current balanced data, $J_k \equiv J = 125$ and

$$\hat{\alpha}_\beta = \frac{.12964}{.12964 + 1.27423 / 125 + 1.75843 / (125)(3)} \cong .897. \text{ (HLM output = .881) There is a little discrepancy here!}$$

3. Measurement Models by Multivariate Three-Level Models

3.1. Multivariate HLM module (HMLM2) descriptive statistics:

LEVEL-1 DESCRIPTIVE STATISTICS					
VARIABLE NAME	N	MEAN	SD	MINIMUM	MAXIMUM
SUBS_SCO	11250	3.69	1.78	0.00	7.00
DSUB1	11250	0.33	0.47	0.00	1.00
DSUB2	11250	0.33	0.47	0.00	1.00
DSUB3	11250	0.33	0.47	0.00	1.00

LEVEL-2 DESCRIPTIVE STATISTICS					
VARIABLE NAME	N	MEAN	SD	MINIMUM	MAXIMUM
X	3750	0.50	0.50	0.00	1.00

LEVEL-3 DESCRIPTIVE STATISTICS					
VARIABLE NAME	N	MEAN	SD	MINIMUM	MAXIMUM
SUBS1_ME	30	3.69	0.41	2.75	4.48
SUBS2_ME	30	3.74	0.45	2.49	4.61
SUBS3_ME	30	3.70	0.44	2.59	4.50
TOTAL_ME	30	11.12	1.15	7.91	13.59
N_STUDEN	30	125.00	48.69	50.00	200.00

3.2. Unrestricted Model (Model 1): Comparable to Table 10.4

a. Fixed effects

	Estimate	Standard error	t-ratio	df	p-value
γ_{000}	3.677	.072	50.888	29	< .001
γ_{100}	.039	.031	1.253	3749	.211
γ_{200}	.044	.030	1.482	3749	.138

b. Variance and covariance components

	Estimate	Standard error
Level 1 & 2		
var(e_{1jk})	3.036	.070
var(e_{2jk})	3.096	.054
var(e_{3jk})	2.966	.069
cov(e_{1jk}, e_{2jk})	1.206	.054
cov(e_{1jk}, e_{3jk})	1.328	.072
cov(e_{2jk}, e_{3jk})	1.290	.054
Level 3		
τ_{β}	.130	.038

Note: The heading “Level 1 & 2” indicates that a vector, e_{jk} , represents the student-level variability of a multivariate outcome, although it was used as the level-2 term in the model equation.

c. Summary

Deviance	# of parameter estimated
42658.678	10

3.3. Heterogeneous level-1 Variance Model (Model 2): Comparable to Table 10.5

a. Fixed effects

	Estimate	Standard error	<i>t</i> -ratio	<i>df</i>	<i>p</i> -value
γ_{000}	3.677	.072	50.848	29	< .001
γ_{100}	.039	.031	1.267	11247	.205
γ_{200}	.044	.030	1.469	11247	.142

b. Variance and covariance components

	Estimate	Standard error
Level 1		
σ_1^2	1.771	.054
σ_2^2	1.866	.056
σ_3^2	1.633	.051
Level 2		
τ_π	1.278	.044
Level 3		
τ_β	.130	.038

c. Summary

Deviance	# of parameter estimated
42664.080	8

3.4. Homogeneous Level-1 Variance Model (Model 3): Comparable to Table 10.6

a. Fixed effects

	Estimate	Standard error	t-ratio	df	p-value
γ_{000}	3.677	.072	50.899	29	< .001
γ_{100}	.039	.031	1.289	11247	.198
γ_{200}	.044	.031	1.445	11247	.148

b. Variance and covariance components

	Estimate	Standard error
Level 1		
σ^2	1.758	.029
Level 2		
τ_{π}	1.274	.044
Level 3		
τ_{β}	.130	.038

c. Summary

Deviance	# of parameter estimated
42664.080	6

3.5. Model Comparisons: Comparable to Table 10.7.

a. Deviance

Model	Parameters Estimated	Deviance
1. Unrestricted model	10	42658.678
2. Heterogeneous level-1 variance model	8	42664.080
3. Homogeneous level-1 variance model	6	42673.165

b. Chi-square difference test

	Chi-square	df	p-value
Model 3 vs. Model 2	9.084	2	.011
Model 2 vs. Model 1	5.402	2	.065
Model 3 vs. Model 1	14.486	4	.006

HGLM Analyses

1. Unconditional Model (1_Uncond.hlm): Comparable to Table 10.8.

a. Fixed effects

item	γ_{00}	se(γ_{00})
Area 1		
1	-1.934	.064
2	-1.415	.069
3	-1.385	.062
4	-0.844	.074
5	-0.556	.056
6	-0.500	.056
7	-0.108	.067
Area 2		
8	-1.897	.079
9	-1.458	.068
10	-1.388	.076
11	-0.754	.067
12	-0.477	.052
13	-0.509	.064
14	-0.071	.057
Area 3		
15	-1.903	.050
16	-1.425	.039
17	-1.452	.049
18	-0.831	.053
19	-0.459	.051
20	-0.448	.053
21*	1.080	.069

* This item was used as the reference item in the model. Therefore, the parameter listed for this item is the estimate of the intercept γ_{00} .

b. Random effects

	Estimate	Standard Error
Level 2		
var(r_{0jk})	.635	.020
Level 3		
var(r_{00k})	.061	.018

2. Conditional Model (2_Cond.h1m): Comparable to Table 10.9.

item	Fixed effects		Random effects	
	γ_{i10}	se	$\text{var}(u_{i1k})$	$\text{cor}(u_{00k}, u_{i1k})$
Area 1				
1	.040	.107		
2	.108	.104		
3	.008	.104		
4	.605*	.126	.141 [†]	.175
5	.150	.104		
6	.107	.105		
7	.097	.107		
Area 2				
8	.101	.106		
9	.032	.104		
10	.073	.104		
11	.559*	.116	.073 [†]	.399
12	.067	.105		
13	.135	.105		
14	.173	.108		
Area 3				
15	.142	.106		
16	.083	.104		
17	.172	.104		
18	.421*	.105	.010	-.745
19	.210	.104		
20	.132	.105		
21	n/a			

* Magnitudes are greater than twice the standard errors.

[†] Magnitudes are significant at $\alpha = .05$ based on chi-square test.

2-level CFA

1. Model with unit factor loadings (Ch10Mplus1.inp)

Chi-Square Test of Model Fit

Value	105.711
Degrees of Freedom	9
P-Value	0.0000

MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Within Level				
MATH_W BY				
AREA1	1.000	0.000	999.000	999.000
AREA2	1.000	0.000	999.000	999.000
AREA3	1.000	0.000	999.000	999.000
Variances				
MATH_W	1.274	0.044	28.847	0.000
Residual Variances				
AREA1	1.758	0.029	61.223	0.000
AREA2	1.758	0.029	61.223	0.000
AREA3	1.758	0.029	61.223	0.000
Between Level				
MATH_B BY				
AREA1	1.000	0.000	999.000	999.000
AREA2	1.000	0.000	999.000	999.000
AREA3	1.000	0.000	999.000	999.000
Intercepts				
AREA1	3.677	0.072	50.886	0.000
AREA2	3.717	0.072	51.435	0.000
AREA3	3.721	0.072	51.498	0.000
Variances				
MATH_B	0.130	0.037	3.468	0.001
Residual Variances				
AREA1	0.000	0.000	999.000	999.000
AREA2	0.000	0.000	999.000	999.000
AREA3	0.000	0.000	999.000	999.000

2. Model with heterogeneous loadings ($\Lambda_W = \Lambda_B$) (Ch10Mplus2.inp)

Chi-Square Test of Model Fit

Value	104.993
Degrees of Freedom	7
P-Value	0.0000

MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Within Level				
MATH_W BY				
AREA1	1.000	0.000	999.000	999.000
AREA2	1.026	0.032	32.276	0.000
AREA3	1.018	0.031	32.863	0.000
Variances				
MATH_W	1.238	0.062	19.902	0.000
Residual Variances				
AREA1	1.758	0.029	61.223	0.000
AREA2	1.758	0.029	61.223	0.000
AREA3	1.758	0.029	61.223	0.000
Between Level				
MATH_B BY				
AREA1	1.000	0.000	999.000	999.000
AREA2	1.026	0.032	32.276	0.000
AREA3	1.018	0.031	32.863	0.000
Intercepts				
AREA1	3.677	0.071	51.541	0.000
AREA2	3.717	0.073	50.917	0.000
AREA3	3.721	0.073	51.316	0.000
Variances				
MATH_B	0.126	0.037	3.443	0.001
Residual Variances				
AREA1	0.000	0.000	999.000	999.000
AREA2	0.000	0.000	999.000	999.000
AREA3	0.000	0.000	999.000	999.000

3. Model with heterogeneous loadings ($\Lambda_W \neq \Lambda_B$) (Ch10Mplus3.inp)

Chi-Square Test of Model Fit

Value	14.882
Degrees of Freedom	4
P-Value	0.0050

MODEL RESULTS

	Estimate	S.E.	Est./S.E.	Two-Tailed P-Value
Within Level				
MATH_W BY				
AREA1	1.000	0.000	999.000	999.000
AREA2	1.004	0.033	30.641	0.000
AREA3	1.002	0.032	31.330	0.000
Variances				
MATH_W	1.282	0.065	19.700	0.000
Residual Variances				
AREA1	1.719	0.028	60.995	0.000
AREA2	1.719	0.028	60.995	0.000
AREA3	1.719	0.028	60.995	0.000
Between Level				
MATH_B BY				
AREA1	1.000	0.000	999.000	999.000
AREA2	1.140	0.231	4.938	0.000
AREA3	1.122	0.223	5.039	0.000
Intercepts				
AREA1	3.678	0.074	49.447	0.000
AREA2	3.730	0.081	46.165	0.000
AREA3	3.706	0.080	46.363	0.000
Variances				
MATH_B	0.098	0.041	2.418	0.016
Residual Variances				
AREA1	0.040	0.010	3.932	0.000
AREA2	0.040	0.010	3.932	0.000
AREA3	0.040	0.010	3.932	0.000

Results Summary: Comparable to Table 10.10.

Parameter	Estimate	Standard Error
λ_{W2}	1.004	.033
λ_{W3}	1.002	.032
λ_{B2}	1.140	.231
λ_{B3}	1.122	.223
σ_W^2	1.719	.028
σ_B^2	0.040	.010
τ_π	1.282	.065
τ_β	0.098	.041