Mathematics in Middle and Secondary School: A Problem Solving Approach

reviewed by Jonathan D. Bostic & Lance Kruse — April 15, 2016

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Mathematics in Middle and Secondary School: A Problem Solving Approach comprises a collection of items stemming from the experiences of authors Alexander Karp and Nicholas Wasserman as mathematics teachers and teacher educators. The purpose of the text is to support secondary inservice mathematics teachers' reflection and growth processes. It would be a useful resource as part of a graduate level, post-baccalaureate, or professional development program. The authors position their book as follows: "[w]hat is needed . . . are special methods courses that would help teachers acquire pedagogical content, knowledge, and skills. It is for the sake, and on the basis, of this kind of course that this book has been written" (p. x).

This review mirrors the format of the book. Part One and Two provide background knowledge about problem solving, explain its role in lesson planning, and emphasize associated theories and implications for teaching mathematics. Part Three explores applications of these theories in educational practices by investigating three content areas: algebra, geometry, and discrete mathematics.

PART ONE: ACTIVATING PRIOR KNOWLEDGE OF TEACHING MATHEMATICS

Teaching is not a formula to follow; it requires managing many factors such as students' prior knowledge, coherence within a lesson and across a unit of study, and the grounding of teaching in theoretical perspectives. Part One reviews these topics as a reminder of the challenges when planning and enacting lessons. For instance, Karp and Wasserman suggest several questions when designing a lesson plan: "What are the basic concepts and ideas studied in this unit? How and when will [an] assessment for the unit be carried out? . . . In what order should the study of the unit be set out?" (p. 36). These questions highlight some of the many complexities of planning mathematics instruction.

PART TWO: WHAT IS PROBLEM SOLVING?

Numerous researchers over the last century have studied problem solving in a variety of contexts (e.g., Dewey, 1937; Lesh & Zawojewski, 2007; Polya, 1945/2004; Schoenfeld, 1987, 1992). In the opening section of Part Two, Karp and Wasserman apply Polya's four step problem solving model to an algebra task.

Various non-vertical straight lines are drawn through point A lying on the ordinate (y) axis above the abscissa (x) axis. Each of these lines intersects the parabola $y = x^2$ at two points. Readers are asked to prove that the product of the abscissas of these two points of intersection is the same for all of the straight lines (p. 72). The authors intend to draw out readers' thinking through the four step process with pedagogical questions using Polya's (1945/2004) framework. This example is useful for the reader to carefully examine on its own, reflect with others, and has utility during coursework, professional development, or self study.

Karp and Wasserman draw heavily on Schoenfeld's work during the era of problem solving (e.g., Schoenfeld, 1987, 1992) and emphasize it as a prominent theme during instruction. Students who routinely engage in problem solving during mathematics instruction are more likely to perceive the connections in the subject and demonstrate greater problem solving performance (Schoenfeld, 1992). There is also discussion about the nuances of problem solving instruction; the process of teaching heuristics (e.g., general strategies for problem solving) is distinct from teaching mathematics and emphasizes problem solving (National Council of Teachers of Mathematics, 2000, 2014).

PART THREE: PROBLEM SOLVING THROUGH SECONDARY MATHEMATICS CONTENT

The third part of *Mathematics in Middle and Secondary School* discusses key ideas when teaching three different mathematical domains: algebra, geometry, and discrete mathematics. Each section is divided into aims, objectives, and analyses of standards in teaching the domain. The section also includes learner difficulties when studying these mathematical domains and examples of algebra lessons. Each domain has strength in improving

teachers' pedagogical content knowledge. Karp and Wasserman provide classroom situated cases fostering problem solving as part of everyday mathematics instruction. These cases help teachers who work within professional development, graduate courses, and professional learning communities reflect on the strengths and challenges of problem solving instruction. More importantly, many cases draw from student teaching vignettes and parallel novice teachers' initial experiences in the classroom. These vignettes also serve as additional professional development opportunities for experienced teachers who mentor novice and student teachers to guide them through the demands of the job. Readers are encouraged to think critically about secondary mathematics instruction using a problem solving lens.

ALGEBRA

Within the algebra chapters, Karp and Wasserman explore the challenges of teaching and learning this mathematical domain (e.g., algebra as a science; patterns, functions, and graphs; and solving equations and inequalities) and they provide in depth information for algebra instruction. The discussion of teaching algebra is rooted in the National Council Teacher of Mathematics' *Principles and Standards* (NCTM, 2000) and the Common Core State Standards (Common Core State Standards Initiative [CCSSI], 2010). Teaching algebra is explored as an interrelated and connected area of study acting as "the gatekeeper to further mathematical study" (p. 177). A considerable amount of*Mathematics in Middle and Secondary School* focuses on the ways i algebra should be taught (e.g., learning algebra as its own language; understanding patterns, relations, and functions; graphical representations; equations; and word problems). The algebra chapters include samples of challenging tasks with follow up discussions regarding why particular tasks are difficult to teach and learn. Research is used to assist the reader in making connections between research and teaching (e.g., Chazan & Yerushalmy, 2003; Howson, 2005; Nemirovsky, 1993; Willoughby, 1997). This chapter concludes with example lessons plans from an algebra teacher and a discussion about lesson plans and their enactment. This assists the reader in assessing lesson plans from mathematical and pedagogical content knowledge perspectives. Due to the important role algebra plays in K–12 mathematics education, the book offers a considerable amount of research, examples, and discussion to further teachers' pedagogical content knowledge.

GEOMETRY

The next set of chapters in *Mathematics in Middle and Secondary School* explores the teaching and studying of geometry. Geometry is rooted in Euclid's *Elements* and has undergone a shift in how it is taught (Karp & Wasserman, 2015). The NCTM (2000) and CCSSI (2010) moved away from Euclidian geometry as a class only for college students and instead identified it as important for all learners. Similar to the algebra chapters, connections between research and practice are made through examinations of extant research (see Clements & Battista, 1992; Nickson, 2000; Van Hiele, 1959, 1986). One idea expressed within this domain is the importance of informal geometry: "it does not presume an acquaintance with axioms and rigorous proofs or with any sort of sophisticated use of mathematical symbolism" (p. 323). The authors discuss how "definitions are usually not starting points of a new idea, but rather the end point" (p. 322 as cited in Van Dormolen & Arcavi, 2000). Students should not memorize definitions but instead create their own working understandings of ideas. A few approaches shared for teaching informal geometry include finding common features of shapes, cutting them, reassembling these shapes, and explaining why a drawing justifies a formula. These approaches shift the focus away from standard formal geometry and introduce learners to informal geometry. Finally, problem solving and lesson planning for a geometry classroom are explored similar to the content of the algebra chapters. A unique feature of these geometry lessons is that both middle and high school lessons are presented and discussions identify both the strengths and weaknesses of the lessons.

DISCRETE MATHEMATICS

The final section of *Mathematics in Middle and Secondary School* explores the teaching and studying of discrete mathematics. Most importantly, the section begins with a description of discrete mathematics (e.g., basic logic, combinatorics, graph theory, matrix algebra, probability and statistics, and set theory) as a means of differentiating this mathematical domain from its counterparts (Bogart, 1991). NCTM's (2000). Data analysis and probability content standards are also referenced to frame the key instructional areas of discrete mathematics. Karp and Wasserman describe the unique role discrete mathematics plays in K–12 curriculum. Although traditional high school mathematics courses do not require discrete math, "many parts of discrete mathematics are integrated into other areas of mathematics" (p. 399) and this type of integration illustrates its interconnectedness (p. 396). The authors highlight key aspects of the structure of mathematics and this is a feature found in the Standards for Mathematical Practice (CCSSI, 2010). This section concludes with an analysis of a variety of discrete mathematics problems and sample lesson plans. This discussion allows for previously considered key ideas to be analyzed in a practical manner.

CONCLUSION

Karp and Wasserman's *Mathematics in Middle and Secondary School* provides an effective framework for exploring the domains of algebra, geometry, and discrete mathematics. The ideas in this book could be deepened with more recent research in problem solving and further exploration in probability and statistics. The first parts provide adequate pedagogical knowledge to better analyze the lesson plans and problem solving ideas presented in subsequent sections. Similarly, the first section of each chapter discusses key ideas, such as aims and objectives, standards, theoretical frameworks, and pedagogical approaches and allows readers to apply these ideas to problem solving and designing lesson plans. These discussions help readers develop a thorough understanding of sound practices in lesson planning and challenges in teaching for, about, and through problem solving.

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